

# The color dipole BFKL-Regge expansion: from DIS on protons to pions to rise of hadronic cross sections

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## Abstract

As noticed by Fadin, Kuraev and Lipatov in 1975 incorporation of asymptotic freedom into the BFKL equation splits the QCD pomeron into a series of isolated poles in complex angular momentum plane. Following our earlier work on the proton structure function we explore the phenomenological consequences of the emerging BFKL-Regge factorized expansion for the small- $x$  structure function of the pion  $F_{2\pi}(x, Q^2)$ . We calculate  $F_{2\pi}$  in a small- $x$  region and find good agreement with the recent H1 determination of  $F_{2\pi}(x, Q^2)$ . We also present the BFKL-Regge factorization based evaluation of the contribution from hard scattering to the observed rise of the  $NN$ ,  $\pi N$  and real photo-absorption  $\gamma N$  and  $\gamma\pi$  total cross sections.

## 1 Introduction

Recently there has been much discussion of the measurement of the pion structure function  $F_{2\pi}(x, Q^2)$  at HERA using deep inelastic scattering (DIS) of high-energy leptons on the virtual pion of the  $\pi N$  Fock state of the proton and selecting semi-inclusive events

$$ep \rightarrow e' n X \quad (1)$$

tagged by a leading neutron [1, 2], for earlier works see [3]. The underlying DIS

$$\gamma(q) + \pi(k) \rightarrow X \quad (2)$$

is considered in the high-energy regime of very large Regge parameter

$$\frac{1}{x} = \frac{W^2 + Q^2}{Q^2} = \frac{1}{x_{Bj}} \gg 1, \quad (3)$$

where  $W^2 = (q + k)^2$  is the  $\gamma^*\pi$  c.m.s. collision energy squared,  $Q^2 = -q^2$  is the virtuality of the photon and  $x$  is the conventional Bjorken variable. For  $Q^2$  much larger than the pion virtuality ( $K^2 = -k^2$ ) the pion can be considered as the non-perturbative on-shell parton in the proton. For the light-cone derivation of the flux of pions in the  $\pi N$  Fock state see [4, 5], a similar flux is found in the recent Regge theory analysis [2]. This makes it possible to measure  $F_{2\pi}(x, Q^2)$  down to  $x \sim 10^{-4}$  way beyond the reach of  $\pi N$  Drell-Yan experiment (hereafter in discussion of DIS we don't make distinction between  $x$  and  $x_{Bj}$ ). Very recently the first experimental data on leading neutron production at HERA and their interpretation in terms of  $F_{2\pi}$  have been reported [6].

Besides giving access to the  $F_{2\pi}$ , the semi-inclusive reaction (1) affects via unitarity the flavor content of the proton sea, for the quantitative interpretation of the recent E866 data [7] see [2], for the review of earlier works see [5].

In the literature there exist several parameterizations of parton distributions in the pion [8, 9] based on the  $\pi N$  Drell-Yan data taken at relatively large  $x \gtrsim 5 \cdot 10^{-2}$ . Extrapolations of these parameterizations to  $x \ll 1$  diverge quite strongly, which is not surprising if one recalls that the conventional DGLAP phenomenology lacks the predictive power at small  $x$  and the small- $x$  extrapolations of the pre-HERA fits to the proton structure functions did generally miss the small- $x$  HERA results [10].

In this communication we address the issue of the small- $x$  behavior of the pion structure function in the BFKL-Regge framework. As noticed by Fadin, Kuraev and Lipatov [11] and discussed in detail by Lipatov [12], the incorporation of asymptotic freedom, i.e. the running QCD coupling, into the BFKL equation makes the QCD pomeron a series of Regge poles. The contribution of the each pole to scattering amplitudes satisfies the standard Regge-factorization [13]. Recently we reformulated the BFKL-Regge expansion in the color dipole (CD) basis [14, 15] which we apply here to calculation of the small- $x$  behavior of the pion structure function. As a by-product of our analysis we present evaluations of the hard scattering contribution to the observed rise of the pion-nucleon, nucleon-nucleon and real photo-absorption  $\gamma N$  and  $\gamma\pi$  total cross sections.

## 2 The color dipole BFKL-Regge factorization

In the color dipole basis the beam-target interaction amplitude is expanded in terms of amplitudes of scattering of color dipoles  $\mathbf{r}$  and  $\mathbf{r}'$  in both the beam ( $b$ ) and target ( $t$ ) particles (here  $\mathbf{r}$  and  $\mathbf{r}'$  are the two-dimensional vectors in the impact parameter plane). Invoking the optical theorem for forward scattering as a fundamental quantity one can use the dipole-dipole cross section  $\sigma(x, \mathbf{r}, \mathbf{r}')$ . The  $\log \frac{1}{x}$  BFKL evolution in the color dipole basis (CD BFKL) has been studied in detail in [14, 15, 16, 17, 18]. Once  $\sigma(x, \mathbf{r}, \mathbf{r}')$  is known one can calculate the total cross section of  $bt$  scattering  $\sigma^{bt}(x)$  making use of the color dipole factorization

$$\sigma^{bt}(x) = \int dz d^2\mathbf{r} dz' d^2\mathbf{r}' |\Psi_b(z, \mathbf{r})|^2 |\Psi_t(z', \mathbf{r}')|^2 \sigma(x, \mathbf{r}, \mathbf{r}'), \quad (4)$$

where  $|\Psi_b(z, \mathbf{r})|^2$  and  $|\Psi_t(z', \mathbf{r}')|^2$  are probabilities to find a color dipole,  $\mathbf{r}$  and  $\mathbf{r}'$  in the beam and target, respectively. Here we emphasize that the dipole-dipole cross section is beam-target symmetric and universal for all beams and targets, all the beam and target dependence is contained in the color dipole distributions  $|\Psi_b(z, \mathbf{r})|^2$  and  $|\Psi_t(z, \mathbf{r})|^2$ .

We start with the minor technical point that  $\sigma(x, \mathbf{r}, \mathbf{r}')$  can depend on the orientation of the target and beam dipoles and can be expanded into the Fourier series

$$\sigma(x, \mathbf{r}, \mathbf{r}') = \sum_{n=0}^{\infty} \sigma_n(x, r, r') \exp(in\varphi) \quad (5)$$

where  $\varphi$  is an azimuthal angle between  $\mathbf{r}$  and  $\mathbf{r}'$ . By the nature of calculation of the beam-target total cross section  $\sigma^{bt}(x)$  only the term  $n = 0$  contributes in (4).

The CD BFKL-Regge factorization uniquely prescribes that the  $1/x$  dependence of the dipole-dipole total cross section  $\sigma(x, r, r')$  is of the form [15]

$$\sigma(x, r, r') = \sum_m C_m \sigma_m(r) \sigma_m(r') \left(\frac{x_0}{x}\right)^{\Delta_m}. \quad (6)$$

Here the dipole cross section  $\sigma_m(r)$  is an eigen-function of the CD BFKL equation

$$\frac{\partial \sigma_m(x, r)}{\partial \log(1/x)} = \mathcal{K} \otimes \sigma_m(x, r) = \Delta_m \sigma_m(x, r), \quad (7)$$

with eigen value (intercept)  $\Delta_m$  and  $\sigma_m(x, r)$  being of the Regge form

$$\sigma_m(x, r) = \sigma_m(r) \left(\frac{x_0}{x}\right)^{\Delta_m}. \quad (8)$$

The running strong coupling exacerbates the well known infrared sensitivity of the CD BFKL equation and infrared regularization is called upon: infrared freezing of  $\alpha_S$  [19] and finite propagation radius  $R_c$  of perturbative gluons were consistently used in our color dipole approach to BFKL equation since 1994 [16]. The past years the both concepts have become widely accepted: for a review of recent works on freezing  $\alpha_S$  see [20], our choice  $R_c = 0.27$  fm has been confirmed by the recent determination of  $R_c$  from the lattice QCD data on the field strength correlators [21].

### 3 Summary on BFKL eigen-functions in the CD basis

Here we recapitulate the principal findings on eigen-functions  $\sigma_m(r)$  and eigenvalues  $\Delta_m$  of the CD BFKL equation [14, 15]. There is a useful similarity to solutions of the Schrödinger equation and the intercept plays the role of a binding energy. The leading eigen-function  $\sigma_0(r)$  for the ground state with largest binding energy  $\Delta_0 \equiv \Delta_{\mathbf{P}}$  is node free. The eigenfunction  $\sigma_m(r)$  for excited state has  $m$  radial nodes. With our infrared regulator the intercept of the leading pole trajectory is found to be  $\Delta_{\mathbf{P}} = 0.4$ . The intercepts  $\Delta_m$  follow closely the law  $\Delta_m = \Delta_0/(m+1)$  suggested first by Lipatov from quasi-classical approximation to running BFKL equation in the related basis. The sub-leading eigen-functions  $\sigma_m(r)$  [14] are also similar to Lipatov's quasi-classical eigen-functions [12] for  $m \gg 1$ . For our specific choice of the infrared regulator the node of  $\sigma_1(r)$  is located at  $r = r_1 \simeq 0.05 - 0.06$  fm, for larger  $m$  the first nodes move to a somewhat larger  $r$  and accumulate at  $r \sim 0.1$  fm, for the more detailed description of the nodal structure of  $\sigma_m(r)$  see [14, 15]. Here we only emphasize that for solutions with  $m \geq 4$  the higher nodes are located at a very small  $r$  way beyond the resolution scale  $1/\sqrt{Q^2}$  of foreseeable DIS experiments. Because for these higher solutions all intercepts  $\Delta_m \ll 1$ , in practical evaluation of  $\sigma^{bt}$  we can truncate expansion

(6) at  $m = 3$  lumping in the term with  $m = 3$  contributions of all singularities with  $m \geq 3$ . Such a truncation can be justified *a posteriori* if such a contribution from  $m \geq 3$  turns out to be a small correction, which will indeed be the case at very small  $x$ .

The practical calculation of  $\sigma(x, r, r')$  by running the CD BFKL evolution requires the boundary condition  $\sigma(x_0, r, r')$  at certain  $x_0 \ll 1$ . The expansion coefficients  $C_m$  in eq.(6) are fully determined by the boundary condition  $\sigma(x_0, r, r')$  and by the choice of the normalization of eigen-functions  $\sigma_m(r)$ . To this end recall that the CD BFKL evolution sums the leading  $\log(1/x)$  diagrams for production of  $s$ -channel gluons via exchange of  $t$ -channel perturbative gluons. It is tempting then, although not compulsory for any fundamental reasons, to take for boundary condition at  $x = x_0$  the Born approximation, i.e. evaluate dipole-dipole scattering via the two-gluon exchange. This leaves the starting point  $x_0$  the sole parameter. We follow the choice  $x_0 = 0.03$  made in [18]. The very ambitious program of description of  $F_2^p(x, Q^2)$  starting from the, perhaps excessively restrictive, but appealingly natural, two-gluon approximation has been launched by us in [14] and met with remarkable phenomenological success [15].

The exchange by perturbative gluons is a dominant mechanism for small dipoles  $r \lesssim R_c$ . In Ref.[18, 15] interaction of large dipoles has been modeled by the non-perturbative, soft mechanism which we approximate here by a factorizable soft pomeron with intercept  $\alpha_{\text{soft}}(0) - 1 = \Delta_{\text{soft}} = 0$ , i.e., flat vs.  $x$  at small  $x$ . Then the extra term  $C_{\text{soft}}\sigma_{\text{soft}}(r)\sigma_{\text{soft}}(r')$  must be added in the r.h.s. of expansion (6). The exchange by two non-perturbative gluons has been behind the parameterization of  $\sigma_{\text{soft}}(r)$  suggested in [18, 33] and used later on in [14] and here, see also Appendix. More recently several related models for  $\sigma_{\text{soft}}(r)$  have appeared in the literature, see for instance models for dipole-dipole scattering via polarization of non-perturbative QCD vacuum [22] and the model of soft-hard two-component pomeron [23].

## 4 CD BFKL-Regge expansion for structure function

Now we recall briefly the formalism for calculation of the target (t) structure function ( $t = p, \pi, \gamma^*, \dots$ ). It is convenient to introduce the eigen structure functions ( $m = \text{soft}, 0, 1, 2, \dots$ )

$$f_m(Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\sigma_m^{\gamma^*}(Q^2), \quad (9)$$

where

$$\sigma_m^{\gamma^*}(Q^2) = \langle \gamma_T^* | \sigma_m(r) | \gamma_T^* \rangle + \langle \gamma_L^* | \sigma_m(r) | \gamma_L^* \rangle. \quad (10)$$

Then the virtual  $\gamma^*t$  photo-absorption cross section and the target structure function  $F_{2t}(x, Q^2)$  take the form ( $m = \text{soft}, 0, 1, 2, \dots$ )

$$\begin{aligned} \sigma^{\gamma^*t}(x, Q^2) &= \sum_m C_m \sigma_m^{\gamma^*}(Q^2) \sigma_m^t \left( \frac{x_0}{x} \right)^{\Delta_m} + \sigma_{\text{val}}^{\gamma^*t}(x, Q^2) \\ &= \sum_m A_m^t \sigma_m^{\gamma^*}(Q^2) \left( \frac{x_0}{x} \right)^{\Delta_m} + \sigma_{\text{val}}^{\gamma^*t}(x, Q^2), \end{aligned} \quad (11)$$

$$F_{2t}(x, Q^2) = \sum A_m^t f_m(Q^2) \left( \frac{x_0}{x} \right)^{\Delta_m} + F_{2t}^{\text{val}}(x, Q^2), \quad (12)$$

where

$$\sigma_m^t = \langle t | \sigma_m(r) | t \rangle = \int dz d^2\mathbf{r} |\Psi_t(z, r)|^2 \sigma_m(r). \quad (13)$$

The color dipole distributions in the transverse (T) and longitudinal (L) photon of virtuality  $Q^2$  derived in [24] read

$$|\Psi_T(z, r)|^2 = \frac{6\alpha_{em}}{(2\pi)^2} \sum_1^{N_f} e_f^2 \{ [z^2 + (1-z)^2] \varepsilon^2 K_1(\varepsilon r)^2 + m_f^2 K_0(\varepsilon r)^2 \} , \quad (14)$$

$$|\Psi_L(z, r)|^2 = \frac{6\alpha_{em}}{(2\pi)^2} \sum_1^{N_f} 4e_f^2 Q^2 z^2 (1-z)^2 K_0(\varepsilon r)^2 , \quad (15)$$

where

$$\varepsilon^2 = z(1-z)Q^2 + m_f^2 . \quad (16)$$

In Eqs. (14)-(16)  $K_0$  and  $K_1$  - are the modified Bessel functions,  $e_f$  is the quark charge,  $m_f$  is the quark mass,  $\alpha_{em}$  is the fine structure constant and  $z$  is the Sudakov variable, i.e. the fraction of photon's light-cone momentum carried by one of the quarks of the pair ( $0 < z < 1$ ). The functional form of  $f_m(Q^2)$  convenient in applications was presented in [15] (see also Appendix). For the practical phenomenology at moderately small  $x$  we include a contribution from DIS on valence quarks in the target  $F_{2\pi}^{\text{val}}(x, Q^2)$  which is customarily associated with the non-vacuum reggeon exchange.

In the evaluation of the proton structure function  $F_{2p}(x, Q^2)$  we use the symmetric oscillator wave function of the 3-quark proton. We recall that in this approximation the proton looks as 3/2 color dipoles spanned between quark pairs. The distribution of sizes of dipoles  $\mathbf{r}$  spanned between quark pairs in the proton reads

$$|\Psi_p(r)|^2 = \frac{1}{2\pi \langle r_p^2 \rangle} \exp \left( -\frac{r^2}{2 \langle r_p^2 \rangle} \right) \quad (17)$$

where  $\langle r_p^2 \rangle = 0.658 \text{ fm}^2$  as suggested by the standard dipole form factor of the proton. In [15] we introduced normalization of  $\sigma_m(r)$  such that for the proton target ( $m = \text{soft}, 0, 1, 2, \dots$ )

$$A_m^p = C_m \sigma_m^p = 1. \quad (18)$$

Within this convention we have the proton expectation values  $\sigma_m^p$  and parameters  $C_m$  of the truncated,  $m = \text{soft}, 0, 1, 2, 3$ , dipole-dipole expansion cited in the Table 1.

We recall that because of the diffusion in color dipole space, exchange by perturbative gluons contributes also to interaction of large dipoles  $r > R_c$  [17]. However at moderately large Regge parameter this hard interaction driven effect is still small. For this reason in what follows we refer to terms  $m = 0, 1, 2, 3$  as hard contribution as opposed to the genuine soft interaction.

Table 1. CD BFKL-Regge expansion parameters.

| $m$  | $\Delta_m$ | $\sigma_m^p$ , mb | $\sigma_m^\pi$ , mb | $C_m$ , $\text{mb}^{-1}$ | $A_m^\pi$ | $A_m^p$ | $\sigma_m^{\gamma^*}(0)$ , $\mu\text{b}$ |
|------|------------|-------------------|---------------------|--------------------------|-----------|---------|--|
| 0    | 0.40       | 1.243             | 0.822               | 0.804                    | 0.661     | 1.      | 6.767                                    |
| 1    | 0.220      | 0.462             | 0.303               | 2.166                    | 0.656     | 1.      | 1.885                                    |
| 2    | 0.148      | 0.374             | 0.244               | 2.674                    | 0.653     | 1.      | 1.320                                    |
| 3    | 0.111      | 0.993             | 0.647               | 1.007                    | 0.651     | 1.      | 3.186                                    |
| soft | 0.         | 31.19             | 18.91               | 0.0321                   | 0.606     | 1.      | 79.81                                    |

## 5 CD BFKL-Regge predictions for $F_{2\pi}(x, Q^2)$

The extension to small- $x$  DIS off pions is quite straightforward. In the normalization (18)

$$A_m^\pi = \frac{\sigma_m^\pi}{\sigma_m^p}. \quad (19)$$

We evaluate  $\sigma_m^\pi = \langle \pi | \sigma_m(r) | \pi \rangle$  with the oscillator approximation for the  $q\bar{q}$  wave function of the pion

$$|\Psi_\pi(r)|^2 = \frac{3}{8\pi \langle r_\pi^2 \rangle} \exp\left(-\frac{3r^2}{8\langle r_\pi^2 \rangle}\right), \quad (20)$$

where the charge radius of the pion suggests  $\langle r_\pi^2 \rangle = 0.433 \text{ fm}^2$ . Then the calculation of  $\sigma_m^\pi$  is parameter-free, the results for  $\sigma_m^\pi$  and  $A_m^\pi$  are cited in Table 1.

The minor change is that in a scattering of color dipole on the pion the effective dipole-dipole collision energy is 3/2 of that in the scattering of color dipole on the three-quark nucleon which is our reference process at the same total c.m.s. energy  $W$ . Then the further evaluation of  $\sigma^{\gamma^*\pi}(x, Q^2)$  and of the pion structure function

$$F_{2\pi}(x, Q^2) = \sum_{m=0}^3 A_m^\pi f_m(Q^2) \left(\frac{3}{2} \frac{x_0}{x}\right)^{\Delta_m} + A_{\text{soft}}^\pi f_{\text{soft}}(Q^2) + F_{2\pi}^{\text{val}}(x, Q^2) \quad (21)$$

shown by the solid curve in fig. 1 does not involve any adjustable parameters. The valence component of the pion structure function (the dot-dashed curve) is quite substantial at  $x \gtrsim 10^{-2}$ , but it is reasonably constrained by the  $\pi N$  Drell-Yan data and can be regarded as weakly model dependent. In our calculations we employ the recent parameterization [9]. We find a good agreement with the recent H1 determination of  $F_{2\pi}(x, Q^2)$  [6]. We do not see any need to modify the above specified natural, perturbative two-gluon exchange, boundary condition to CD BFKL evolution.

Although the agreement with the experimental data is good, testing the complete CD BFKL-Regge expansion in all its complexity is not a trivial task. As we mentioned in the Introduction, small- $x$  extrapolations of existing DGLAP fits [8, 9] to the  $\pi N$  Drell-Yan data diverge quite strongly, but the proper fine tuning of the input can produce the DGLAP fits which in a limited range of moderately small  $x$  will be very similar to our results. To this end, several aspects of the predictive power of CD BFKL approach are noteworthy. First, there is an obvious prediction that the leading hard pole contribution to single-pomeron exchange will dominate at sufficiently small  $x$  for all  $Q^2$ , but it is not easy to check it within the reach of the HERA experiments and better understanding of the unitarity corrections to single-pomeron exchange is needed at extremely small  $x$  beyond the HERA range. Second, as noticed in [17] because of the short propagation radius for perturbative gluons,  $R_c^2 \ll \langle r_p^2 \rangle, \langle r_\pi^2 \rangle$ , in our CD BFKL approach we have an approximate additive quark counting for hard components,  $m = 0, 1, 2, 3$ , i.e.,

$$\frac{\sigma_m^p}{\sigma_m^\pi} \approx \frac{3}{2}, \quad (22)$$

see entries in Table 1. Third, according to the conventional estimates for the proton and pion radii based on the experimental data on charge radii, the dipoles spanned between quark pairs in the proton, and the quark and antiquark in the pion, have approximately

similar sizes, cf. (17) and (20). Consequently, for a reason quite different from that for the hard BFKL exchange, an approximate additive quark counting holds also for the soft-pomeron exchange, see Table 1. A good approximation to the CD BFKL-Regge factorization prediction is

$$F_{2\pi}(x, Q^2) \simeq \frac{2}{3} F_{2p}\left(\frac{2}{3}x, Q^2\right). \quad (23)$$

This is an arguably natural, but far from trivial, relationship. Given that the origin of additive quark counting for hard and soft components is so different, it is not surprising that (23) has not necessarily been borne out by the diversity of the pre- and post-HERA DGLAP fits to the pion and proton structure functions. Fourth, the finding of a substantial soft contribution shown by the short-dashed line must not surprise anyone in view of a familiar strong sensitivity of the results of DGLAP evolution to the input structure function at a semi-hard starting point. Fifth, the CD BFKL approach predicts uniquely, that sub-leading eigen structure functions  $f_{m \geq 1}(Q^2)$  have their first node at  $Q^2 \sim 20 - 60 \text{ GeV}^2$ , see [14, 15] and Appendix, and we would like to comment on this node effect in more detail.

The point is that in the vicinity of the node of sub-leading contributions the pion structure function is well reproduced by the Leading Hard+ Soft+Valence Approximation (LHSVA)

$$F_{2\pi}^{\text{LHSVA}}(x, Q^2) \simeq \frac{2}{3} \left[ f_0(Q^2) \left( \frac{3x_0}{2x} \right)^{\Delta_0} + f_{\text{soft}}(Q^2) \right] + F_{2\pi}^{\text{val}}(x, Q^2), \quad (24)$$

which gives a unique handle on the intercept  $\alpha_{\mathbf{P}}(0) = 1 + \Delta_0$  of the leading hard BFKL pole. This point is illustrated in fig. 1 where the dotted curve represents the LHSVA. A comparison with the solid curve for the complete BFKL-Regge expansion shows that, as a matter of fact, the contribution from sub-leading BFKL-Regge poles is marginal at all values of  $Q^2$  of the practical interest. In order to delineate the impact of the valence component we show by the long-dashed curve the sum of the soft and leading hard pole contributions without the valence component. Even such a crude two-pole model does a good job at  $x \lesssim (2.5) \cdot 10^{-3}$ , although at larger  $x$  the impact of the valence component on the  $x$ -dependence of the pion structure function is substantial. The accuracy of the LHSVA improves rapidly at larger  $Q^2$ , see the boxes for  $Q^2 = 7.5$  to  $13.3$  to  $28.6$  and  $100 \text{ GeV}^2$  in fig. 1. As a manifestation of the nodal structure of  $f_{m \geq 1}(Q^2)$  the LHSVA overshoots the result of the full complete-Regge expansion slightly at  $Q^2 = 28.6$  and  $100 \text{ GeV}^2$ , while it undershoots the complete BFKL-Regge expansion slightly at  $Q^2 \leq 13.3 \text{ GeV}^2$ .

As eq. (23) suggests, this discussion of LHSVA is fully applicable to the proton structure function as well. This point is demonstrated by the decomposition into different components of our earlier CD BFKL-Regge predictions for the proton structure function [14, 15] shown in Fig. 2. Because the LHSVA formula (24) is sufficiently simple and the soft contribution is small for  $Q^2 \sim 10\text{-}100 \text{ GeV}^2$ , a reliable determination of  $\alpha_{\mathbf{P}}(0)$  for the rightmost BFKL singularity is feasible even from the pion data. An independent determination of  $\alpha_{\mathbf{P}}(0)$  and, consequently, the test of the CD BFKL approach is possible from the charm structure function of the proton - as we argued elsewhere [27], it receives a negligible soft and sub-leading contributions and is entirely dominated by the contribution from the rightmost BFKL singularity (for the related recent discussion of the charm structure function see also [28]).

## 6 Extending CD BFKL-Regge expansion to real photons and hadrons

Finally, with certain reservations on absorption corrections we can extend the BFKL-Regge factorization from DIS to real photo-production and even hadron-hadron scattering. Take for instance pion-nucleon scattering. There is always a contribution from small-size dipoles in the pion to the color dipole factorization formula (6). For example, a probability  $w_\pi(r < r_0)$  to find dipoles of size  $r \lesssim r_0$  can be estimated as

$$w_\pi(r < r_0) \sim \frac{3}{8} \frac{r_0^2}{\langle r_\pi^2 \rangle}, \quad (25)$$

which gives quite a substantial fraction of the pion,

$$w_\pi(r < 0.2 \text{ fm}) \sim 3 \cdot 10^{-2} \quad (26)$$

the interaction of which with the target nucleon proceeds in the legitimate hard regime typical of DIS. The corresponding contribution to  $\sigma_{\text{tot}}^{\pi N}$  must exhibit the same rapid rise with energy as the proton structure function. Furthermore, in the BFKL approach there is always a diffusion in the dipole size by which there is a feedback from hard region to interaction of large dipoles and vice versa. How substantial is such a hard contribution to the hadronic and real photon total cross sections?

The explicit realization of this idea within the CD BFKL-Regge factorization is embodied in expansion for the vacuum exchange contributions to the total cross sections

$$\sigma^{pp} = \sum_m A_m^p \sigma_m^p \left( \frac{2}{3} \frac{x_0}{x} \right)^{\Delta_m}, \quad \sigma^{\gamma p} = \sum_m A_m^p \sigma_m^{\gamma*}(0) \left( \frac{x_0}{x} \right)^{\Delta_m}, \quad (27)$$

$$\sigma^{\pi p} = \sum_m A_m^\pi \sigma_m^p \left( \frac{x_0}{x} \right)^{\Delta_m}, \quad \sigma^{\gamma \pi} = \sum_m A_m^\pi \sigma_m^{\gamma*}(0) \left( \frac{3}{2} \frac{x_0}{x} \right)^{\Delta_m}, \quad (28)$$

where  $m = \text{soft}, 0, 1, 2, 3$ . In the real photo-production limit the light quark masses  $m_{u,d} = 0.135 \text{ GeV}$ ,  $m_s = 0.285 \text{ GeV}$  serve to define the transverse size of the hadronic component of the photon which it is reasonable to expect close to the transverse size of vector mesons and/or pions. The real photo-absorption cross sections  $\sigma_m^{\gamma*}(0)$  and  $\sigma_{\text{soft}}^{\gamma*}(0)$  as given by eqs.(9,32) extended to  $Q^2 = 0$  are cited in Table 1. Regarding the similarity of the transverse size of real photon and  $\rho/\pi$ -meson, notice that the ratio  $\sigma_{\text{soft}}^{\gamma^* p} / \sigma_{\text{soft}}^{\pi p} \sim 1/230$  is very close to the standard vector meson dominance estimate [29].

In eqs.(27,28) the plausible choice of the Regge parameter is  $1/x = W^2/m_\rho^2$ , in the NN and  $\gamma\pi$  scattering we introduce the aforementioned energy rescaling factor  $2/3$  and  $3/2$ , respectively. In our approach  $\Delta_{\text{soft}} = 0$  and  $\sigma_{\text{soft}} = \text{const}(W^2)$ , so that the intrusion of hard regime into soft scattering is the sole source of the rise of total cross sections. The CD BFKL-Regge factorization based evaluation of the vacuum exchange contribution to  $pp$ ,  $\pi p$ ,  $\gamma p$  and  $\gamma\pi$  total cross sections [30] is presented in Fig. 3. At low energies the vacuum exchange contribution underestimates the observed total cross section which can evidently be attributed to the non-vacuum Regge exchanges not considered here.

Although the hard pomeron component is important in both the purely hadronic ( $NN$ ,  $\pi N$ ) and photo-absorption reactions, there is an important distinction between the two cases.



Namely, in contrast to the proton/pion wave function (17,20) which is smooth at  $r \rightarrow 0$ , the real photon wave function squared  $|\Psi_T(r)|^2$  is singular (14). Furthermore, this singularity is a legitimate pQCD effect and makes (i) evaluation of hard contributions,  $m = 0, 1, 2, 3$ , to  $\gamma\pi, \gamma p$  cross sections more reliable and (ii) uniquely predicts that hard contribution to  $\sigma_{\text{tot}}^{\gamma\pi}, \sigma_{\text{tot}}^{\gamma p}$  is relatively stronger than to  $\sigma_{\text{tot}}^{\pi p}, \sigma_{\text{tot}}^{pp}$ . Indeed, a closer inspection of the Table 1 shows that in  $\gamma N$  the relative weight of hard component  $\sum_0^3 \sigma_m / \sigma_{\text{soft}}$  is almost twice as large as in the  $\pi N$  scattering.

We observe that hard effects in both the  $pp, \pi p$  and  $\gamma\pi, \gamma p$  to exhaust to a large extent, or even completely, the observed rise of  $\sigma_{\text{tot}}(W^2)$  at moderately large  $W^2$ . Furthermore, there arises an interesting scenario, discussed first in a simplified version in [31], in which soft interactions at moderately high energies can be described by the soft pomeron with  $\Delta_{\text{soft}} = 0$  plus small contribution from hard scattering, the combined effect of which mimics the usually discussed phenomenological pomeron pole endowed with the intercept  $\Delta \simeq 0.08$  [32]. In this scenario the real issue is not an explanation of the rise of hadronic  $\sigma_{\text{tot}}(W^2)$  at moderately high energies, rather it is whether there exists a mechanism to tame a too rapid rise of extrapolation of the hard component of the CD BFKL-Regge expansion to very high-energies. To this end recall that we considered only the non-unitarized running CD BFKL amplitudes too rapid a rise of which must be tamed by the unitarity absorption corrections. The discussion of the issue of unitarization goes beyond the scope of the present communication.

## 7 Conclusions

We explored the consequences for small- $x$  structure functions and high energy total cross sections from the color dipole BFKL-Regge factorization. We use very restrictive perturbative two-gluon exchange as a parameter-free boundary condition for BFKL evolution in the color dipole basis. Under plausible assertions on the color dipole structure of the pion, our parameter-free description of the pion structure function agrees well with the H1 determinations. The found relationship between  $F_{2p}(x, Q^2)$  and  $F_{2\pi}(x, Q^2)$  is very close to  $F_{2\pi}(x, Q^2) \simeq \frac{2}{3} F_{2p}(\frac{2}{3}x, Q^2)$ . The contribution of sub-leading BFKL-Regge poles to the pion and proton structure function is found to be small in a broad range of  $Q^2$  of the practical interest, which makes feasible the determination of the intercept  $\alpha_{\mathbf{P}}(0)$  for the rightmost BFKL singularity from the structure function data.

Because of the diffusion in color dipole space, the hard scattering which is a dominant feature of  $F_{2\pi}(x, Q^2)$  and  $F_{2p}(x, Q^2)$  at large  $Q^2$  contributes also to real photo-absorption and hadron-hadron scattering. This make plausible a scenario in which the rising hard component and the genuine soft component with  $\Delta_{\text{soft}} = 0$  mimics the effective vacuum pole with  $\Delta_{\text{soft}} \simeq 0.08$ .

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## Appendix

Although we have certain ideas on the shape of eigen-functions  $\sigma_m(r)$  as a function of  $r$  and/or eigen structure functions  $f_m(Q^2)$  as a function of  $Q^2$  [14], they are only available as a numerical solution to the running color dipole BFKL equation. On the other hand, for the practical applications it is convenient to represent the results of numerical solutions for  $f_m(Q^2)$  in an analytical form

$$f_0(Q^2) = a_0 \frac{R_0^2 Q^2}{1 + R_0^2 Q^2} \left[ 1 + c_0 \log(1 + r_0^2 Q^2) \right]^{\gamma_0}, \quad (29)$$

$$f_m(Q^2) = a_m f_0(Q^2) \frac{1 + R_0^2 Q^2}{1 + R_m^2 Q^2} \prod_{i=1}^{n_{max}} \left( 1 - \frac{z}{z_m^{(i)}} \right), \quad m \geq 1, \quad (30)$$

where  $\gamma_0 = \frac{4}{3\Delta_0} = \frac{10}{3}$  and

$$z = \left[ 1 + c_m \log(1 + r_m^2 Q^2) \right]^{\gamma_m} - 1, \quad \gamma_m = \gamma_0 \delta_m \quad (31)$$

and  $m_{max} = \min\{m, 2\}$ .

The nodes of  $f_m(Q^2)$  are spaced by 2-3 orders of magnitude in  $Q^2$ -scale. The first nodes of sub-leading  $f_m(Q^2)$  are located at  $Q^2 \sim 20 - 60 \text{ GeV}^2$ , the second nodes of  $f_2(Q^2)$  and  $f_3(Q^2)$  are at  $Q^2 \simeq 5 \cdot 10^3 \text{ GeV}^2$  and  $Q^2 \simeq 2 \cdot 10^4 \text{ GeV}^2$ , respectively. The third node of  $f_3(Q^2)$  is at  $\sim 2 \cdot 10^7 \text{ GeV}^2$ , way beyond the reach of accelerator experiments at small  $x$ . The parameters tuned to reproduce the numerical results for  $f_m(Q^2)$  at  $Q^2 \lesssim 10^5 \text{ GeV}^2$  are listed in the Table 2. For  $m = 3$  in this limited range of  $Q^2$  we take a simplified form with only two first nodes, it must not be used for  $Q^2 \gtrsim 10^5 \text{ GeV}^2$ .

Table 2. CD BFKL-Regge structure functions parameters.

| $n$  | $a_m$  | $c_m$  | $r_m^2, \text{ GeV}^{-2}$ | $R_m^2, \text{ GeV}^{-2}$ | $z_m^{(1)}$ | $z_m^{(2)}$ | $\delta_m$ |
|------|--------|--------|---------------------------|---------------------------|-------------|-------------|------------|
| 0    | 0.0232 | 0.3261 | 1.1204                    | 2.6018                    |             |             | 1.         |
| 1    | 0.2788 | 0.1113 | 0.8755                    | 3.4648                    | 2.4773      |             | 1.0915     |
| 2    | 0.1953 | 0.0833 | 1.5682                    | 3.4824                    | 1.7706      | 12.991      | 1.2450     |
| 3    | 0.4713 | 0.0653 | 3.9567                    | 2.7756                    | 1.4963      | 6.9160      | 1.2284     |
| soft | 0.1077 | 0.0673 | 7.0332                    | 6.6447                    |             |             |            |

The soft component of the proton structure function derived from eq.(9) with  $\sigma_{\text{soft}}(r)$  taken from [33] is parameterized as follows

$$f_{\text{soft}}(Q^2) = \frac{a_{\text{soft}} R_{\text{soft}}^2 Q^2}{1 + R_{\text{soft}}^2 Q^2} \left[ 1 + c_{\text{soft}} \log(1 + r_{\text{soft}}^2 Q^2) \right], \quad (32)$$

with parameters cited in the Table 2.

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## Figure Captions

- Fig.1** Predictions from the CD BFKL-Regge factorization for the pion structure function  $F_{2\pi}(x_{Bj}, Q^2)$  (solid lines). The experimental data from the H1 Collaboration [6] are shown by full circles. The different components of  $F_{2\pi}(x_{Bj}, Q^2)$  are shown: the valence contribution (dashed-dotted), the non-perturbative soft contribution (dashed), the Leading Hard+Soft+Valence Approximation  $F_{2\pi}^{\text{LHSVA}}(x_{Bj}, Q^2)$  (dotted) and the sum of the soft and leading hard pole contributions without valence component (long dashed).
- Fig.2** The decomposition into different components of earlier predictions [14, 15] from the CD BFKL-Regge factorization for the proton structure function  $F_{2p}(x, Q^2)$  (solid lines). The experimental data from the H1 and ZEUS Collaborations [25, 26] are shown by circles and triangles, respectively. The different components of  $F_{2p}(x, Q^2)$  are shown: the valence contribution (dashed-dotted), the non-perturbative soft contribution (dashed), the Leading Hard+Soft+Valence Approximation (LHSVA)  $F_{2p}^{\text{LHSVA}}(x, Q^2)$  (dotted) and the sum of the soft and leading hard pole contributions without valence component (long dashed).
- Fig.3** The CD BFKL-Regge factorization evaluation of the vacuum exchange contribution to  $\sigma_{\text{tot}}^{NN}$  (solid line in box (a)),  $\sigma_{\text{tot}}^{\pi N}$  (solid line in box (b)),  $\sigma_{\text{tot}}^{\gamma N}$  (solid) and  $\sigma_{\text{tot}}^{\gamma\pi}$  (dashed line in box (c)). The data points are from [30].

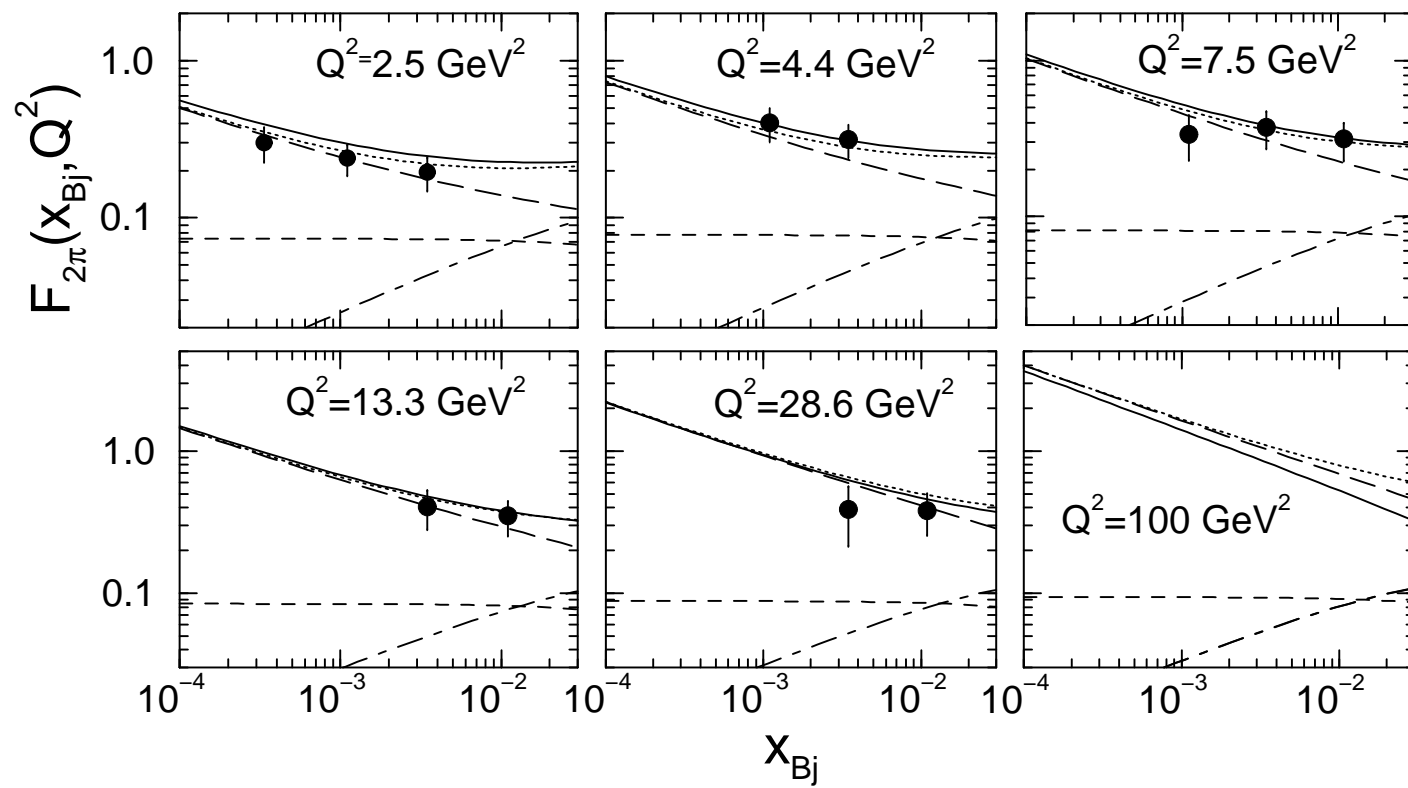


Fig. 1

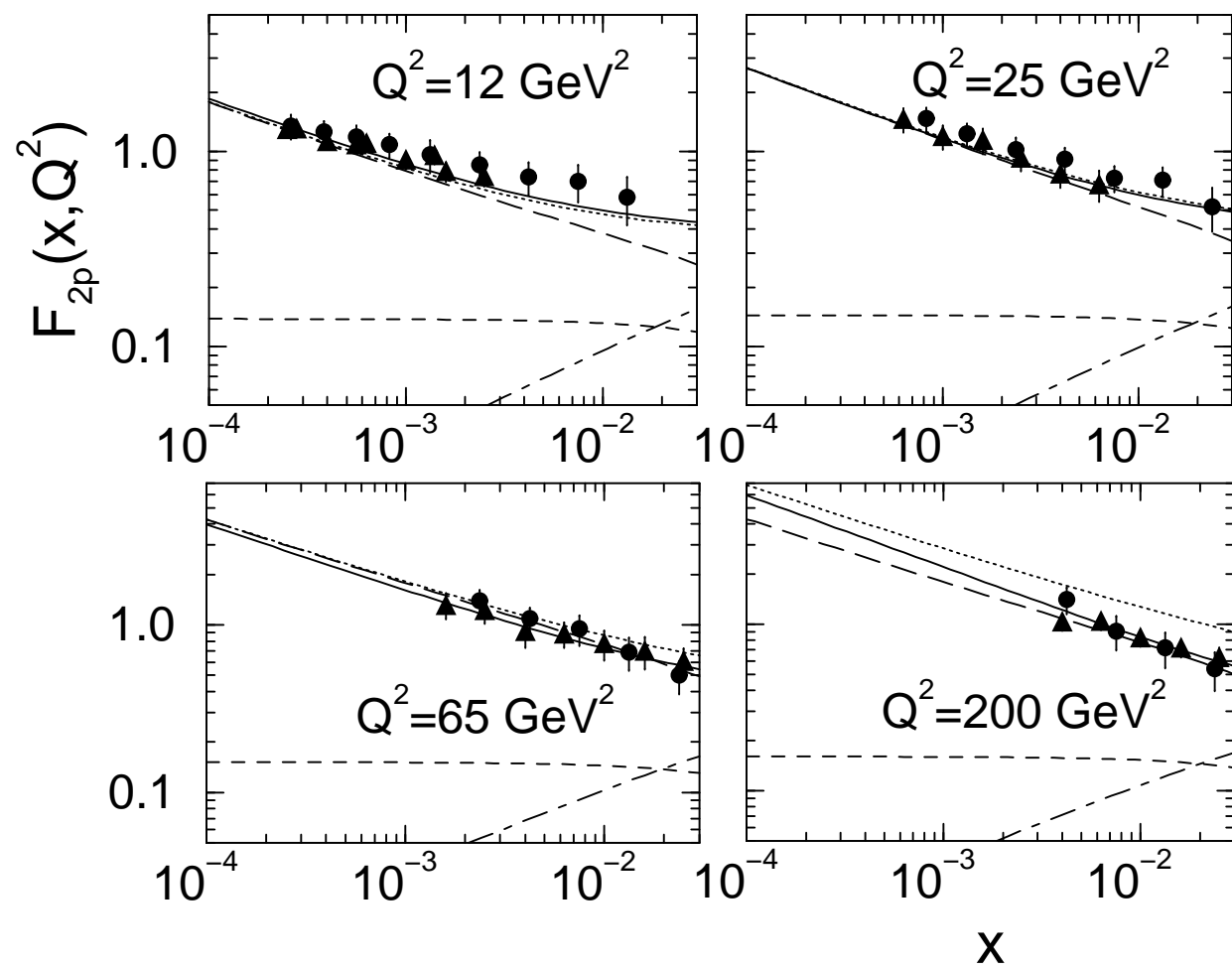


Fig. 2

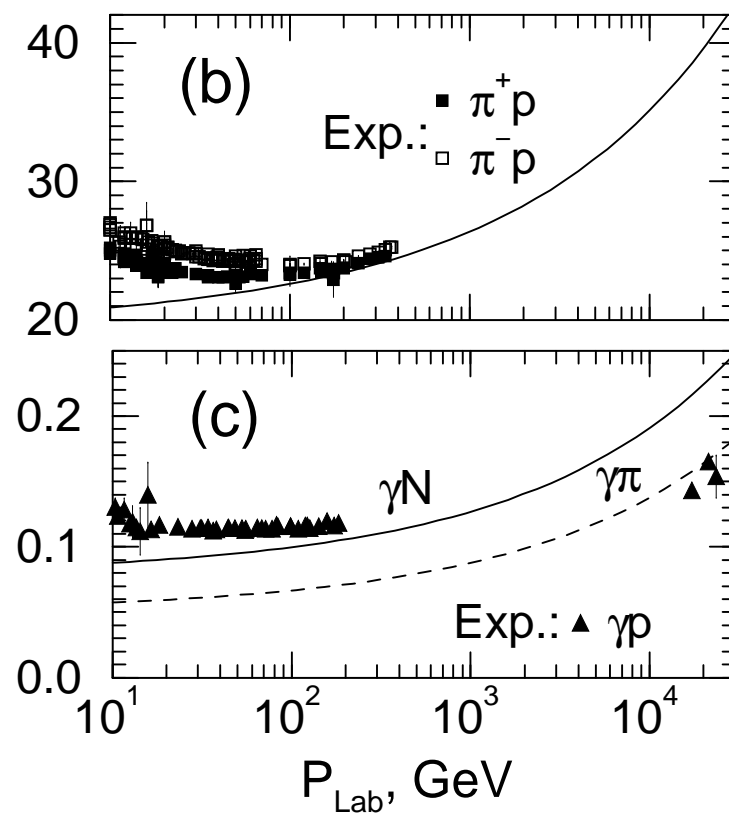
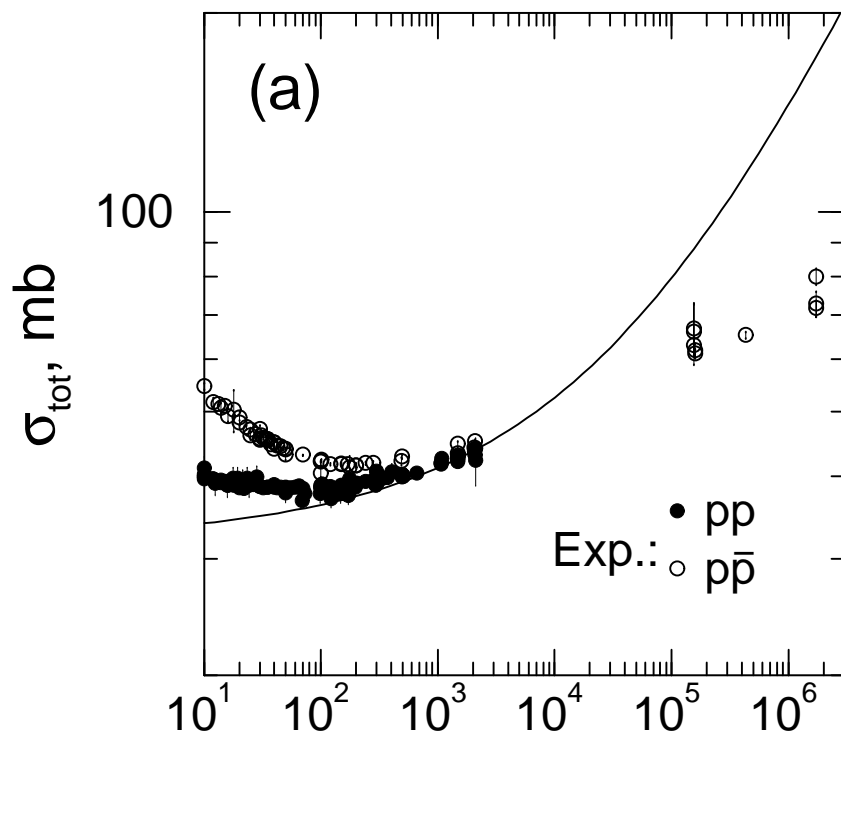


Fig. 3